Infinite Dimensional Analysis, Quantum Probability
and Related Topics
Vol. 15, No. 3 (2012) 1250014 (18 pages)

DOI: 10.1142/S0219025712500142

ON SUMMABLE, POSITIVE POISSON–MEHLER KERNELS BUILT OF AL-SALAM–CHIHARA AND RELATED POLYNOMIALS

PAWEŁ J. SZABŁOWSKI

Department of Mathematics and Information Sciences,
Warsaw University of Technology,
ul. Koszykowa TS, 00-662 Warsaw, Poland
pawel.szablowski@gmail.com

Received 22 February 2012
Published 2 October 2012
Communicated by Izumi Kubo

Using special technique of expanding ratio of densities in an infinite series of polynomials orthogonal with respect to one of the densities, we obtain simple, closed forms of certain kernels built of the so-called Al-Salam–Chihara (ASC) polynomials. We consider also kernels built of some other families of polynomials such as the so-called big continuous q-Hermite polynomials that are related to the ASC polynomials. The constructed kernels are symmetric and asymmetric. Being the ratios of the densities they are automatically positive. We expand also reciprocals of some of the kernels, getting nice identities built of the ASC polynomials involving six variables like e.g., formula (3.6). These expansions lead to asymmetric, positive and summable kernels. The particular cases (referring to $q = 1$ and $q = 0$) lead to the kernels build of certain linear combinations of the ordinary Hermite and Chebyshev polynomials.

Keywords: q-Hermite; big continuous q-Hermite; Al-Salam–Chihara; Chebyshev polynomials; Mercier; Poisson–Mehler kernels; summability and positivity of kernels; bilinear generating functions.

AMS Subject Classification: Primary: 33D45, 05A30; Secondary: 42A16, 60E05

1. Introduction

In many models of the so-called q-oscillators considered in quantum physics, classical and noncommutative probability or generally in some branches of analysis appears a problem of summing and examining positivity of kernels built of certain families of orthogonal polynomials (see e.g., Refs. 8, 9 and 1). The kernels (more precisely the Poisson–Mehler kernels) are, generally speaking, expressions of the form $K(x,y) = \sum_{n\geq 0} a_n D_n(x) F_n(y)$, where $\{D_n\}_{n\geq 0}$, $\{F_n\}_{n\geq 0}$ are certain families of orthogonal polynomials and $x, y, \{a_n\}_{n\geq 0}$ are real numbers. Usually the numbers a_n are of the form $t^n/\|D_n\|\|F_n\|$, $|t| < 1$ where $\|\cdot\|$ denotes certain (usually L_2)