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SYMPLECTIC T_7, T_8 SINGULARITIES AND LAGRANGIAN TANGENCY ORDERS

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Abstract We study the local symplectic algebra of curves. We use the method of algebraic restrictions to classify symplectic T_7, T_8 singularities. We define discrete symplectic invariants (the Lagrangian tangency orders) and compare them with the index of isotropy. We use these invariants to distinguish symplectic singularities of classical T_7 singularity. We also give the geometric description of symplectic classes of the singularity.

Keywords: symplectic manifold; curves; local symplectic algebra; algebraic restrictions; relative Darboux theorem; singularities

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1. Introduction

In this paper we study the symplectic classification of singular curves under the following equivalence.

Definition 1.1. Let N_1 and N_2 be germs of subsets of symplectic space $(\mathbb{R}^{2n}, \omega)$. N_1 and N_2 are *symplectically equivalent* if there exists a symplectomorphism germ

$$\Phi\colon (\mathbb{R}^{2n}, \omega) \to (\mathbb{R}^{2n}, \omega)$$

such that $\Phi(N_1) = N_2$.

We recall that ω is a symplectic form if ω is a smooth non-degenerate closed 2-form, and $\Phi\colon \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ is a symplectomorphism if Φ is diffeomorphism and $\Phi^*\omega = \omega$.

Symplectic classification of curves was first studied by Arnold. In [**2**] he discovered new symplectic invariants of singular curves. He proved that the A_{2k} singularity of a planar curve (the orbit with respect to standard $\mathcal{A}$-equivalence of parametrized curves) split into exactly $2k+1$ symplectic singularities (orbits with respect to symplectic equivalence of parametrized curves). He distinguished different symplectic singularities by different orders of tangency of the parametrized curve to the *nearest* smooth Lagrangian submanifold. He posed the problem of expressing these invariants in terms of the local algebra's

