Jednoczesne metody znajdowania wartości szczególnych oraz zer wielomianów ortogonalnych

On Simultaneous Rootfinding Methods for Singular Values and Orthogonal Polynomials

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Treść: Rozważamy zastosowania pewnych metod wyznaczania miejsc zerowych w problemie obliczania wartości szczególnych macierzy dwudiagonalnych.
Proponujemy algorytmy będące modyfikacjami metod klasycznych: Weierstrassa, Abertha i Bairstowa obliczania wszystkich pierwiastków wielomianu. Wykorzystywane są własności rozpatrywanych macierzy zarówno w konstrukcji samego algorytmu jak i odpowiednim doborze wartości początkowych oraz w wyborze warunku zakończenia obliczeń. Rozważane zmodyfikowane metody mogą być również stosowane do wyznaczania pierwiastków wielomianów ortogonalnych.

Słowa kluczowe: metody jednoczesnego wyznaczania pierwiastków, wartości szczególne macierzy, wartości własne macierzy, wielomiany ortogonalne.

Abstract: We consider applications of certain rootfinding methods for the bidiagonal singular value problem. The problem of computing singular values of a bidiagonal n-by-n matrix is equivalent to computing eigenvalues of a symmetric tridiagonal $2n$-by-$2n$ matrix.
The algorithms we propose are modifications of the classical Weierstrass, Aberth and Bairstow methods for computing all roots of a polynomial. We make use of the properties of the matrix, both in algorithms themselves and in the choice of the initial approximation and the stopping criterion. We also apply these modified methods to finding roots of orthogonal polynomials.

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Key words: Simultaneous rootfinding methods, singular values, eigenvalues, orthogonal polynomials.

1. Introduction

In this paper we present several modifications of existing rootfinding algorithms. They may be used for solving several kinds od problems such as finding singular values of matrices, eigenvalues of certain matrices, roots of some orthogonal polynomials.
The algorithms that we consider here are the Weierstrass method (see e.g. [16, 14, 11]), the Aberth method ([1]) and the simultaneous version of the Bairstow method ([10, 13]).
We will start by presenting the details of these problems. Then we will describe the algorithms and their modifications. These modifications can be applied to every simultaneous rootfinding algorithm.

2. Singular values and rootfinding methods

In this section we will describe how rootfinding methods can be used to compute the singular values of any matrix.

Let $A \in \mathbb{R}^{n\times n}$ have the singular values $\sigma_1 \geqslant \cdots \geqslant \sigma_n$. Recall that they are the square roots of the eigenvalues of the matrix A^TA. However, methods based on forming A^TA and computing its eigenvalues may lead to significant loss of accuracy, especially in small singular values.
On the contrary, the problem of computing singular values of bidiagonal matrices is very well conditioned in the sense of the relative error. By a bidiagonal matrix we mean a matrix with nonzero entries located only on the main diagonal and on the superdiagonal. Demmel and Kahan [4] (see also [2, p.90]) proved that all singular values of bidiagonal matrices may be computed with high relative accuracy.
The method we propose here is a variation of the approach first outlined by Golub and Kahan in [7]. They used the fact that any matrix $A \in \mathbb{R}^{n\times n}$ can be decomposed as

$$Q^TAP = B = \begin{pmatrix} c_1 & b_2 & & & \\ & c_2 & b_3 & & \\ & & \ddots & \ddots & \\ & & & c_{n-1} & b_n \\ & & & & c_n \end{pmatrix},$$

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